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# Human networks and toxic relationships

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## **Abstract**

We devise a theoretical model to shed light on the dynamics leading to the so called toxic relationships. We want to investigate what policy interventions people could advocate to protect themselves and to reduce suffocant assuefaction so to escape to the trap of physical or psychological abuses either in family or at work. By assuming that the toxic partner's behavior is exogenous and that the main source of addiction is income or wealth, and solving a dynamical system of differential equations we find that an asymptotically stable equilibrium with positive love is always possibile for enough high level of appealing unless subsidies to reduces assuefaction are introduced. Also the existence of a third uncondicionally reciprocating part as a benckmark (which represent not only the real presence of another partner but also the support from family, friends and overall private organizations in helping victims of domestic abuses or private organizations that offer economic and psychological support as well as legal counseling to victims of bullying at workplace and placement offices which e ectively help to find soon another job)plays an important role in reducing the toxic partner's appealing. By solving our model we outline the condition for a best mixed policy where both monetary subsidies and offering alternatives are at work.

**Keywords:** Dynamical systems, stability, economics, relationships, networks  
**JEL Classification Numbers:** C62, D11, D91

## **1 Introduction**

Economics is a social science which analyzes the behavior of firms that maximize their profit and individuals who maximize their utility measured as happiness within the context

of society. Therefore many interactions that are not traditionally considered economic are often investigated. A very particular recently developed branch of economics literature is the Economics of Love focused on the household's well-being, marriage and long-term relationships alongside to their effects on the economic growth (for an exhaustive survey on this literature see Browning et al,2014; Grossbard,2015).

The first pioneristic work in this new field of Economics appeared in the early 1970s with the theory of marriage by Becker (1973), according to which each individual wants to find a partner with whom they will maximise their own well-being. The equilibrium in this marriage market is reached when no person can change her partner or become single and at the same time to experience an higher well being (referred to the consumption of household commodities). Becker's empirical analysis shows that the gain from marriage is positively dependant on income, relative difference in their wage rates, and the level of intangible variables such as education and beauty. Moreover, individuals choose partners with similar traits such as height, race, social background, etc. By including among the factors which can affect marriages also love (defined as a situation where the utility of one individual depends on the commodity consumption of their partner as well as their own), Becker (1974) finds that it raises the likelihood of two people marrying because their well being is likely to be higher.

After this pioneristic works, many other analyses have been mainly focused on to increase our understanding of how households operate. Outcomes of interest concern consumption, savings, labor supply and other uses of time, household formation and dissolution, demand for health and other forms of human capital, fertility and children outcomes, demand for environmental quality, migration, and household produced goods(for a detailed survey are Browning et al,2014 and Grossbard,2015).

Many papers also investigate the determinants of long-lasting relationships and their effects on economics development. Among them, Brines and Joyner (1999) conducted an empirical study on the stability of marriage and cohabiting couples in the United States. They have found that inequality in employment and income among the cohabiting couple increase the chances of separation, although the effect is not symmetric because inequality has a larger impact when the female partner earns more than the male partner. Grossbard(2015) studies family as a complex decision unit where partners with potentially different objectives make decisions about consumption, work and fertility. Couples marry and divorce partly based on their ability to coordinate these activities. More recently Johnson et al.(2018) applied a unique longitudinal approach to study the long-term outcomes of relationships among 3,405 couple pairs. This report's main questions concerned predictors of relationship longevity, from measures of conflict frequency, types of behaviors experienced during times of conflict, satisfaction with the relationship, and whether partners believed their relationships would last or not. The German study's findings show most importantly that complacency is perhaps the most significant trap to avoid. Working to avoid falling into that same-old same- old routine, and your relationship can be vital and fulfilling for years to come. While there is a certain number of studies on marriage and cohabiting couples, on our knowledge there are only few papers which theoretically study the economic dynamics of romance. Rinaldi (1998) proposed a mathematical model based on a linear dynamical system where three aspects of love dynamics are taken into account: the forgetting process (oblivion), the

pleasure of being loved (return), and the reaction to the appeal of the partner (instinct). The results of the model show that the system turns out to be positive if the appeals of the two individuals are positive. Sprott J.C. (2001), in his paper presented to the Chaos and Complex system seminar in Madison, extends those results and discusses various models of love and happiness, like the model of romance between Romeo and Juliet and that about triangular relationship to analyse how a third part affects the stability of a relationship. Wauer et.al (2007) studie human romantic relationships via system dynamics methodology where a non-linear modeling is proposed and analyzed, showing that there are short- and long-termed fluctuations of personal feelings due to, for instance, biological cycles and varying stresses from the daily job. The variability is expected to be more limited for couples of cautious individuals. In Regan (2008) individuals behave in order to optimize their net benefit from a given relationship. The desire to maximize their rewards and minimize their costs, where both change over time, and the inequity of the benefits to contributions among the individuals in a relationship will cause unhappiness. This clearly also depends on the role of expectations (Thibaut and Kelly, 1959) and on the level of investment that is given to the relationship. This commitment then determines whether or not the relationship will be maintained (Rusbult,1983). Satsangi and Shimano(2012) add to the initial model of Rinaldi (1998) a crossed interactions among partners to analyse the effects and sinergies of learning and adaptation from living togheter. According to the results of this model the system is asymptotically stable if the ratio of appeals is greater than the reciprocal of the ratio of mutual intensiveness coefficient. In this paper we aim to give an additional contribution to the literature in Economics of Love by devising a slightly modified version of the model in Rinaldi (1998) to shed light on the dynamics leading to the so called toxic relationships. We also want to investigate what policy interventions people could advocate to protect themselves and to reduce suffocant assuefaction so to escape to the trap of physical or psychological abuses either in family or at work. The dynamics of couples' relationships are analyzed trough a system of differential equations representing the laws of motions of the amount of love that two individuals put in a relationship which also depends on any source of partner's appeal (financial, physical, intellectual,coparenthood,etc.). This appeal in our work is assumed to change over time proportionally both to the effect of the other partner's love and to a variable representing the main source of addiction (for instance, wealth,status,physical appearence,etc). The aim of our analysis is to build a model that takes into account the case where toxic relationships are at work, that is,when a partner can decide to stay in a relationship despite the low or null love from the other, just because of the partner's appealing that makes more difficult to split. Therefore we focus on situations where love is transformed in a negative dependance and the relationship produces the same effect like a drogue generating dangerous addiction.

By toxic relationship we mean a relationship disorder which can have many forms but all are characterized by a disparity, a non-egalitarian situation in which one of the two subjects depends on the other, triggering a mechanism of dominance and subjection. One puts in the relationship much more affect than the other, whose contribution can even be zero. Unlike a healthy relationship where we can still cut out our spaces, maintaining a capacity for self-determination, benefit from reciprocity, in a toxic relationship instead the emotional dependance enters into play that makes the partner our exclusive interlocutor, so that being

happy and enthusiastic depends exclusively on the other, just like in drug addiction. In order to avoid the abandonment and the consequent lack of affection, the addicted-partner cancels himself, while the counterpart, who probably has the idea of a relationship in which love is obtained by putting aside one's ego, exploits the relationship only to feel admired and to exercise control. This type of relationship implies psychological violence but can also physical and result in tragic episodes of murders mainly against women.

Another unfortunately typical example of toxic relationship is that of bullying at a workplace, that probably happens even more when the person engaged in mobbing is the boss, and the silence of victims and spectators end up to favor it. The reason for this behavior is obviously the fear of being involved, having retaliation of some kind or even losing the job. In our work we highlight the conditions for the possibility that a poisoning tight arises and pushes for long-term relationships, relatively stable, caused by addiction. Due to the assuefaction, the partner's behavior still seems appealing regardless his low reciprocating affection. In Section 2 we give an analytical definition of a toxic relationship by solving an intertemporal dynamic model, where the toxic partner's behaviour is assumed to be exogenous. By assuming that the main source of addiction is income or wealth, we find that an asymptotically stable equilibrium with positive love is always possible for enough high level of appealing unless subsidies to reduce assuefaction are introduced. In Section 3 we compare two alternative policies that can be adopted to heal from addiction either through a subsidy that can reduce the partner's appeal and then we introduce a third unconditionally reciprocating part as a benchmark which represents an alternative, but less attractive, to the partner's love, but who plays an important role in reducing the toxic partner's appealing. It substantially mimics not only the real presence of another partner but also the support from family, friends and overall private organizations in helping victims of domestic abuses to recover their full life. It may also represent private organizations that offer economic and psychological support as well as legal counseling to victims of bullying at workplace and placement offices which effectively help to find soon another job. By solving our model we outline the condition for a best mixed policy where both monetary subsidies and offering alternatives are at work. Section 4 contains our Discussions.

## 2 Basic Model of a toxic relationship

In this Section we devise a slightly modified version of the model by [9] where the level of love an individual puts in a relationship varies over the time depending either on oblivion and the partner's love but also on her appealing which in turn follows a specific law of motion. We assume for simplicity that the partner's appealing decreases at the same rate of love (i.e. the oblivion's rate is the same) and decreases as higher it is the difference between the amount of love he puts in the relationship and that of partner, in each period of time. We also assume that it depends on a constant factor of appealing like income, personality, physical appearance, etc. If we denote by 1,2 the submitted and the toxic partner respectively, so that  $0 < x_2 < x_1$ , therefore we have to study the stability of the

following system of dynamic equations:

$$\begin{cases} \dot{x}_1(t) = -\alpha x_1(t) + \beta \bar{x}_2 + \gamma_1 A_1(t) \\ \dot{A}_{11}(t) = -\alpha A_1(t) + k(x_1 - \bar{x}_2) + M_2 \end{cases} \quad (1)$$

where

- i)  $x_1(t)$  is the love, which consists of both passion and intimacy as well as monetary support, individual one has for the partner at time  $t$ .
- ii)  $\bar{x}_2$  is the love the partner puts in the relationship and that we assume, in this first simplified version of the model, is an exogenous variable of the system and remains constant over time.
- iii)  $A_1(t)$  is the appeal factor. It is a subjective variable, and is dependent on the perception of the partner.
- iv)  $M_2$  measures the source of partner's appealing that for simplicity we assume is constant over time.
- v)  $\alpha$  refers to the forgetting coefficient and measures how quickly the state of love will decrease, exponentially, in the absence of the partner.
- vi)  $\beta$  refers to the response of each player's love to their partner's love level.
- vii)  $k$  measures the sensitivity of an individual with respect to the excessive unreciprocated love he gives to the partner.
- viii)  $\gamma_1$  refers to the response coefficients to the appeal of the other partner.

We assume that the coefficients  $\bar{x}_2, M_2, \alpha, \beta, k, \gamma_1 \in (0, 1]$ .

**Definition 1.** Assume it exists a minimum level of desired love  $\wedge x_2$  for individual 1 and  $\bar{x}_2 < \wedge x_2$  but  $\frac{M_2}{\bar{x}_2} > k$  define this situation a toxic relationship.

The above Definition state a toxic relationship since individual 1 still loves 2 even if this last doesn't reciprocate (neither at the minimum possible level normally requested). This happens because attraction is very high. It strongly depends on the values of parameters and this condition is always satisfied for low levels of  $k$  and  $\alpha$ , and as higher is  $\gamma_1$ .

Steady state equilibria

**Proposition 2.** If  $\sqrt{k\gamma_1} < \alpha < 1$ ,  $M_2 \geq k\bar{x}_2$  and  $\beta > \alpha$  then

$$x_1^* = \frac{\alpha\beta - k(\gamma_1 + 1)}{\alpha^2 - k\gamma_1} \bar{x}_2 + \frac{M_2}{\alpha^2 - k\gamma_1}$$

and

$$A_1^* = \frac{\alpha(M_2 - k\gamma_1 \bar{x}_2) + \beta k \gamma_1}{\alpha^2 - k\gamma_1}$$

is an asymptotically stable equilibrium for system (1).

*Proof.* Let  $A$  be the coefficient matrix of the system

$$A = \begin{pmatrix} -\alpha & \gamma_1 \\ k & -\alpha \end{pmatrix}.$$

since  $\det A > 0$  and  $\text{tr} A < 0$  the conclusion follows.

Remark that, even if  $\bar{x}_2 = 0$  the first individual still loves the second one since  $x_1^* = \frac{M_2}{\alpha^2 - k\gamma_1}$

If individual 1 starts loving individual 2 at time  $t = 0$ , he will love her forever ( $\tilde{A}$  "sempre crescente o a campana o concava?"). This is true also when  $\bar{x}_2$  is very low or equal to zero

### 3 Healing

In this Section we look for conditions to help the submitted partner to heal from toxicity by reducing the toxic partner's appeal via a subsidy  $s \in [0, 1]$ .

**Definition 3.** *Individual 1 overcomes a toxic relationship, i.e. 1 is healed, if  $x^* = 0$ .*

In system (1) we introduce the subsidy  $s$  in the dynamic of  $A_1$

$$\begin{cases} \dot{x}_1(t) = -\alpha x_1(t) + \beta \bar{x}_2 + \gamma_1 A_1(t) \\ \dot{A}_{11}(t) = -\alpha A_1(t) + k(x_1 - \bar{x}_2) + M_2(1 - s) \end{cases} \quad (2)$$

**Proposition 4.** *If  $k$  is taken such that*

$$\frac{\alpha\beta}{\gamma_1 + 1} < k < \frac{\alpha\beta}{\gamma_1 + 1} + \frac{M_2}{(\gamma_1 + 1)\bar{x}_2} \quad \text{if} \quad \frac{\alpha\beta}{\gamma_1 + 1} + \frac{M_2}{(\gamma_1 + 1)\bar{x}_2} < \frac{\alpha\beta}{\gamma_1}$$

and

$$\frac{\alpha\beta}{\gamma_1 + 1} < k < \frac{\alpha\beta}{\gamma_1} \quad \text{if} \quad \frac{\alpha\beta}{\gamma_1 + 1} + \frac{M_2}{(\gamma_1 + 1)\bar{x}_2} \geq \frac{\alpha\beta}{\gamma_1}$$

then the subsidy  $s = \frac{\alpha\beta - k(\gamma_1 + 1)}{M_2} \bar{x}_2 + 1$  is healing individual 1.

*Proof.* It is easy to see that in this case

$$x_{1s}^* = \frac{\alpha\beta - k(\gamma_1 + 1)}{\alpha^2 - k\gamma_1} \bar{x}_2 + \frac{M_2(1 - s)}{\alpha^2 - k\gamma_1}$$

hence the result follows from the above assumptions. Another way to heal might be the presence of a third person, denoted by 3, we want to see if 1 can heal from the second partner. System (1) becomes

$$\begin{cases} \dot{x}_1(t) = -\alpha x_1(t) + \beta(\bar{x}_2 - x_3) + \gamma_1 A_1(t) \\ \dot{A}_{11}(t) = -\alpha A_1(t) + k(x_1 - \bar{x}_2 + x_3) + M_2 - M_3 \end{cases} \quad (3)$$

where  $x_3$  is the amount of love of individual 3 towards individual 1 and  $M_3$  with  $M_3 < M_2$  is his appealing.

**Proposition 5.** *If (mettere condizioni per cui  $x_3$  sia positivo)*

$$x_3 = \frac{M_2 - M_3}{\alpha\beta - k(\gamma_1 + 1)} + \overline{x_2}$$

*then 3 is healing individual 1.*

*Proof.* It is enough to see that in this case

$$x_{13}^* = \frac{\alpha\beta - k(\gamma_1 + 1)}{\alpha^2 - k\gamma_1}(\overline{x_2} - x_3) + \frac{M_2 - M_3}{\alpha^2 - k\gamma_1}$$

We remark that if in addition to the second partner there is also a subsidy, then proceeding as before

$$x_{13s}^* = \frac{\alpha\beta - k(\gamma_1 + 1)}{\alpha^2 - k\gamma_1}(\overline{x_2} - x_3) + \frac{M_2(1 - s) - M_3}{\alpha^2 - k\gamma_1}$$

and

$$s_3 = \frac{\alpha\beta - k(\gamma_1 + 1)}{M_2}(\overline{x_2} - x_3) + 1 - \frac{M_3}{M_2}$$

is healing individual 1.

## 4 Conclusions

A toxic relationship can be defined as a relationship characterized by behaviors of the toxic partner that are emotionally and frequently physically damaging to the other. A healthy relationship involves mutual caring, respect, compassion, a strong interest in the partners' happiness and in the couples both share control and decision-making. On the contrary, a toxic relationship is characterized by insecurity, self-centeredness, dominance and control. Anyway, when two individuals have a toxic relationship, we usually look at the behaviors of the toxic partner, but we must look equally hard at the individual who is the recipient of the toxic behavior. In fact, according to psychologists we should also investigate why an adult stays in a relationship that will almost inevitably damage him or her emotionally or physically. We think that this often happens because addiction may play a very important role, implying that the partners' appeal grows over time regardless to the amount of love the other puts in that relationship.

Probably people with low self esteem are the most subjected to this kind of addiction and often psychotherapy for couples is needed. In the worst cases when toxicity externalizes in physical violence also more objective policy solutions are necessary. A typical case of violent toxic behaviour is that of domestic abuses, which is a major issue in the US and around the world, and many nonprofit organizations work tirelessly to provide critical support and services to victims. Every year, more than 10 million men and women in the US are subjected to Domestic Violence. Its impact can be felt far and wide. More than 1 in 3 women (35.6%) and more than 1 in 4 men (28.5%) in the U.S. report having experienced rape, physical violence, and/or stalking by an intimate partner in their lifetime. Another



example of a toxic relationship, unfortunately very widespread, is that of bullying at the workplace, where too often abuse and harassment are considered normal events, so that the workers who suffer them prefer to be silent. This happens especially when their work is precarious, whose duration and conditions depend on the discretion of the employer. Istat data for Italy tell us that 9 out of 100 women during their working life have been subjected to harassment or blackmail with a sexual background at work (1 million 403 thousand), but that only 20% talk about it with someone (usually office colleagues) and only 0.7% complaint, for fear of retaliation, shame, or for a distorted sense of guilt, slander. The damages are not only for the worker but also for the company, in terms of lower productivity, increased risk of accidents and conflict. The inevitable repercussions then fall on the health service (treatments, drugs) and on the social security system (illnesses, injuries, etc.). In this paper we aimed to approach to these issues from a theoretical point of view with the purpose to devise an analytical model which can be used to highlight the main points of this problem and investigate useful policy solutions. To this purpose we used the model of Rinaldi (1998) where we add the possibility that the toxic partner's appealing evolve dynamically according to a specific law of motion. In our model we assume that this law depends on oblivion as well as on the excess of love with respect to the partner and a constant variable measuring the values of the source of addiction (for instance income or wealth, etc.). Our model shows that in the most simple case where the partner's behaviour is given (exogenous) and hopelessly toxic, an asymptotically stable equilibrium, with a submitted partner always in love even when not enough reciprocated, is always possible for high values of the addiction. Nevertheless an opportune measure of correction based on subsidies can be introduced. On the otherwise side, the lackness of low levels of government help is often one of the main reasons for the low levels of immediately reported domestic abuses. For instance, in Italy According to GROVIO (Group of Experts on Action against Violence against Women and Domestic Violence and Extreme Consequences) of the European Council about 80% of abuse happens at home and there not enough dormitory to host other 5000 women who have left their house to escape from abusive partners. Public funds are not entire and a fairly used to this purpose, only 0,02% of the available sources have been spent to support victims and create structures to host them. Similmente to reduce dependence on boss or coworkers and can incentivize the victim to denounce are specific laws, penalties and high compensation for the monetary damage but also the biological and moral one, union organizations offering cheap or free legal or help, support for finding other jobs and reintegration, etc.

To take into account this problem, we look for an alternative policy and study how the results of our theoretical model change when an alternative third part enters the game by competing with the toxic partner for the love of the submitted one. In this case lower or zero subsidies can be necessary as it works like a substitute to decrease the toxic dependence. Anyway a mix of policy intervention based both in subsidies and help from third part can be realized. Therefore when a policy based only on subsidies is not always sufficient other factors can be at work to rescue from toxicity. Into reality, often subsidies are not enough high or people need also to preserve their dignity, security, counseling, etc. and don't want to live with free lunches all the time. In our model for instance a high source of addiction may come from also by a very low self esteem which can instead be raised given victims other opportunities like alternative jobs or legal/counseling support. This is why on pur

opinion the best solutions relies on a mix of policies where not only the government helps with subsidies but also private organizations can offer support to the victims of abuse as well placement offices can help in finding new jobs.

An alternative, very interesting policy solution, under this point's view, for instance, is that of the Italian government to support poor people, living under the level of subsistence according to the Eurostat definition, or suffering from structural unemployment. This policy implies both a guaranteed small income of citizenship, to subsidize the consumption of the minimal necessary goods, as well as a crew of experts who help them for free with the job search so they can find a stable position in the long run and subsidies are not necessary anymore. With this work we aimed to give a contribution to the literature on Economics of Love and in particular we hope to incentive more theoretical and empirical studies on relationships which can be harmful and devise better policy solutions. In particular we think that a mix of policies, directly or indirectly created to fight this phenomenon, like the Italian policy about the income of citizenship (whose effects on that could be tested empirically only in few years) based on monetary help and supportive institutions and organizations could be very effective.

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